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M.A./M.Sc (Second Semester)

EXAMINATION, MAY-JUNE, 2022

STATISTICS

Paper-I

(Linear Algebra)

Time : Three Hours] *[Maximum Marks:80*

Note: Attempt all sections as directed.

Section-A

(Objective/Multiple Choice Questions)

(1 mark each)

Note : Attempt all questions.

Chose the correct answer:

P.T.O.

1. A set of linear equations is represented by the matrix equation Ax = b. The necessary condition for the existence of a solution for this system is :-

   (A) A must be invertible.

   (B) B must be linearly independent of the columns of A.

   (C) B must be linearly dependent of the columns of A.

   (D) None of these.

2. If rank (A) = 2 and rank (B) =3 then rank (AB) is :-

   (A) 5

   (B) 6

   (C) 3

   (D) Data inadequate

3. For what values of λ, do the simultaneous equation 2x + 3y = 1, 4x + 6y = λ have finite solutions?

   (A) $\lambda = 0$

   (B) $\lambda = 1$

   (C) $\lambda = 2$

   (D) $\lambda \neq 0$

4. The condition for which the Eigen values of the matrix $\begin{bmatrix} 2 & 1 \\ 1 & k \end{bmatrix}$ are positive is :

(A) $k > \frac{1}{2}$

(B) $k < -\frac{1}{2}$

(C) $k > -2$

(D) $k > 0$

5. The rank of the matrix (m × n) where m < n cannot be more than

(A) m

(B) n

(C) m × n

(D) n - m

6. If A and B are square matrices of the same order, then tr (AB) =

(A) tr (A + B)

(B) tr (A) + tr (B)

(C) tr (A) tr (B)

(D) tr (BA)

7. If A and B are symmetric matrices of same order, then

(A) AB is always symmetric

(B) AB is skew-symmetric

(C) AB is never symmetric

(D) AB is symmetric if and only if AB = BA

8. Find the sum of the Eigen values of the matrix

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 0 & 4 & 6 \\ 5 & 2 & 1 \end{bmatrix}$$

(A) 7

(B) 8

(C) 9

(D) 10

9. Consider the following two statements

I The maximum number of linearly independent column vectors of a matrix A is called the rank of A.

II If A is an n × n square matrix, it will be non singular is rank (A) = n. with reference to the above statements, which of the following applies?

(A) Both the statements are true.

(B) Both the statements are false.

(C) Statement I is true but statement II is false.

(D) Statement II is true but statement I is false.

10. Ring $(\mathrm{Q}, +, \bullet)$, $(\mathrm{R}, +, \bullet)$ and $(\mathrm{C}, +, \bullet)$ where Q, R and C are rational real and complex numbers are

(A) Field

(B) Normal Subgroup

(C) Group

(D) None of these

11. A ring $(\mathrm{R}, +, \bullet)$ with identity is said to be.

(A) Normal

(B) Ring

(C) Division ring

(D) None of these

12. Every field is

(A) Group

(B) Semi-group

(C) Integral domain

(D) None of these

13. A ring R is called Bolean Ring if for “x belong to R”

(A) $x^2 = x$

(B) $x^2 = e$

(C) $x = e$

(C) None of these

14. Number of automorphism from $Q\left(\sqrt{2}\right)$ to $Q\left(\sqrt{3}\right)$ is.

(A) 1

(B) 2

(C) 3

(D) 0

15. The set R consisting of single element 0 with two binary operations defined by 0 + 0 = 0 and 0.0=0 is a ring. This ring is called.

(A) identity ring

(B) null ring

(C) integral domain

(D) None of these

16. The number of elements in a finite group is called the .... of the group.

    (A) size

    (B) spectrum

    (C) order

    (D) none of these

17. If the dimensions of subspaces W_1 and W_2 of a vector W are respectively 5 and 7 and $\dim(W_1+W_2)=1$ then dim $(W_1 \cap W_2)$ is

    (A) 5

    (B) 7

    (C) 9

    (D) 11

18. Let $T : R^2 \to R^2$ be a linear transformation such that T ((1, 2)) = (2, 3) and T ((0, 1)) = (1, 4). Then T ((5, - 4)) is

    (A) (6, - 1)

    (B) (- 6, 1)

    (C) (- 1, 6)

    (D) (1, - 6)

19. Cancellation laws are always satisfied in -

    (A) Integral domain

    (B) Ring

    (C) Commutative ring

    (D) Ring with unit element

20. Let L/K be a finite extension of fields. Which of the following assertions is correct?

    (A) If the characteristic of K is zero, then L/K is normal.

    (B) If the characteristic of K is zero, then L/K is separable.

    (C) If L/K is normal, then L/K is a Galois extension.

    (D) If the characteristic of K is positive, then L/K is normal if and only if it is separable.

Section- B

(Very Short Answer Type Questions)

(2 marks each)

Note : Attempt all questions.

1. Define Ring and Ring automorphism.

2. Define Field and polynomial over the field.

3. Define Principal Ideal Domain.
4. Let $T: V \to W$ be a linear transformation between vector spaces. Then define the kernel of T $(ie. \ker T)$.
5. Define scalar matrix and non-singular matrix.
6. What is the canonical form of a matrix?
7. Define Vector space.
8. What are the properties of vector addition?

Section C

(Short Answer Type Questions)

(3 marks each)

Note : Attempt all questions.

1. Prove that any finite integral domain is field.
2. Prove that if an integral domain I not possessing the unity clemet is of characteristic $p \neq 0$, then $px = 0 \; \forall x \in \mathrm{I}$, and if m < p then $mx \neq 0$ if $x \neq 0$.
3. Define orthogonal matrices and give its properties.
4. Find the zeros of the polynomial.

$$f(x) = x^4 + 2x^3 - 21x^2 - 22x + 40$$

if they are in A.P.

5. If a determinant A has two identical columns or two identical rows then prove that |A| = 0.
6. Let $A = \begin{bmatrix} \sqrt{2} & 1 \\ 0 & \sqrt{2} \end{bmatrix}$. Find all Eigen vectors of A. Is A diagonalizable i.e. does there exist an invertible S such that $S^{-1}AS$ is diagonal. Justify you answer.
7. Define row-reduced echelon matrix and give its example.
8. Prove that a non-empty subset W of V is a subspace of V if and only if for each pair of Vector α, β in W and each scalar c in F the vector $C\alpha + \beta$ is again in W.

Section D

(Long Answer Type Questions)

(5 Marks each)

Note : Attempt all questions.

1. Let A, B, C be matrices with proper dimensions. Then prove that matrix multiplication is distributive with respect to matrix addition i.e.

   A (B + C) = AB + AC.

OR

Show that every square matrix is uniquely expressible as the sum of a symmetric matrix and a skew symmetric matrix.

2. Show that the matrix $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ satisfies Coley-Hamilton theorem.

OR

Define cyclic group and give its properties.

3. Find the rational zeros and the decomposition of the polynomial

$f(x) = 4x^4 + 8x^3 + 7x^2 + 8x + 3$

Over the field of rational numbers.

OR

Prove that if a set S of vectors spans a finite-dimensional vector space V(f), there exists a subset of S which forms a basis of V.

4. Use the Gram-Schmidt process to find an orthonormal basis of the subspace

W = span {[1 1 0], [1 1 1]} of R_3.

OR

If *f* is a homomorphism or linear transformation of *U(f)* into *V(f),* then *f(0) = 0*

i.e their zero vectors correspond, and

$f(-\alpha) = -f(\alpha), \alpha \in \cup$.